

# The matching energy of graphs with given edge connectivity\*

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## Abstract

Let  $G$  be a simple graph of order  $n$  and  $\mu_1, \mu_2, \dots, \mu_n$  the roots of its matching polynomial. The matching energy of  $G$  is defined as the sum  $\sum_{i=1}^n |\mu_i|$ . Let  $K_{n-1,1}^k$  be the graph obtained from  $K_1 \cup K_{n-1}$  by adding  $k$  edges between  $V(K_1)$  and  $V(K_{n-1})$ . In this paper, we show that  $K_{n-1,1}^k$  has maximum matching energy among all connected graph with order  $n$  and edge connectivity  $k$ .

Keywords: Matching energy, Edge connectivity, Graph energy, Matching

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## 1 Introduction

We use Bondy and Murty [2] for terminology and notations not defined in this paper and consider undirected and simple graphs only. Let  $G = (V, E)$  be such a graph with order  $n$ . Denote by  $m(G, t)$  the number of  $t$ -matchings of  $G$ . Clearly,  $m(G, 1) = e(G)$ , the size of  $G$ , and  $m(G, t) = 0$  for  $t > \lfloor n/2 \rfloor$ . It is both consistent and convenient to define  $m(G, 0) = 1$ .

Recall that the *matching polynomial* of a graph  $G$  is defined as

$$\alpha(G) = \alpha(G, \lambda) = \sum_{t \geq 0} (-1)^t m(G, t) \lambda^{2t}$$

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and its theory is well elaborated [3–5].

The eigenvalues  $\lambda_1, \lambda_2, \dots, \lambda_n$  of the adjacency matrix  $A(G)$  of  $G$  are said to be the eigenvalues of the graph  $G$ . The *energy* of  $G$  is defined as

$$E(G) = \sum_{i=1}^n |\lambda_i|. \quad (1.1)$$

The theory of graph energy is well developed nowadays, for details see [6, 7, 16]. The Coulson integral formula [10] plays an important role in the study on graph energy, its version for an acyclic graph  $T$  is as follows:

$$E(T) = \frac{2}{\pi} \int_0^{+\infty} \frac{1}{x^2} \ln \left[ \sum_{t \geq 0} m(T, t) x^{2t} \right] dx. \quad (1.2)$$

Motivated by formula (1.2), Gutman and Wagner [11] defined the *matching energy* of a graph  $G$  as

$$ME = ME(G) = \frac{2}{\pi} \int_0^{+\infty} \frac{1}{x^2} \ln \left[ \sum_{t \geq 0} m(G, t) x^{2t} \right] dx. \quad (1.3)$$

Energy and matching energy of graphs are closely related, and they are two quantities of relevance for chemical applications, for details see [1, 8, 9].

The following result gives an equivalent definition of matching energy.

**Definition 1.1** [11] *Let  $G$  be a graph of order  $n$ , and let  $\mu_1, \mu_2, \dots, \mu_n$  be the roots of its matching polynomial. Then*

$$ME(G) = \sum_{i=1}^n |\mu_i|. \quad (1.4)$$

The formula (1.3) induces a *quasi-order* relation over the set of all graphs on  $n$  vertices: if  $G_1$  and  $G_2$  are two graphs of order  $n$ , then

$$G_1 \preceq G_2 \Leftrightarrow m(G_1, t) \leq m(G_2, t) \text{ for all } t = 0, 1, \dots, \lfloor \frac{n}{2} \rfloor. \quad (1.5)$$

If  $G_1 \preceq G_2$  and there exists some  $i$  such that  $m(G_1, i) < m(G_2, i)$ , then we write  $G_1 \prec G_2$ . Clearly,

$$G_1 \prec G_2 \Rightarrow ME(G_1) < ME(G_2).$$

Recall that the *Hosoya index* of a graph  $G$  is defined as  $Z(G) = \sum_{t \geq 0} m(G, t)$  [12]. So we also have that

$$G_1 \prec G_2 \Rightarrow Z(G_1) < Z(G_2).$$

The following result gives two fundamental identities for the number of  $t$ -matchings of a graph [4, 5].

**Lemma 1.2** *Let  $G$  be a graph,  $e = uv$  an edge of  $G$ , and  $N(u) = \{v_1(=v), v_2, \dots, v_j\}$  the set of all neighbors of  $u$  in  $G$ . Then we have*

$$m(G, t) = m(G - uv, t) + m(G - u - v, t - 1), \quad (1.6)$$

$$m(G, t) = m(G - u, t) + \sum_{i=1}^j m(G - u - v_i, t - 1). \quad (1.7)$$

From Lemma 1.2, it is easy to get the following result.

**Lemma 1.3** [11] *Let  $G$  be a graph and  $e$  one of its edges. Let  $G - e$  be the subgraph obtained from  $G$  by deleting the edge  $e$ . Then  $G - e \prec G$  and  $ME(G - e) < ME(G)$ .*

By Lemma 1.3, among all graphs on  $n$  vertices, the empty graph  $E_n$  without edges and the complete graph  $K_n$  have, respectively minimum and maximum matching energy [11]. It follows from Eqs. (1.2) and (1.3) that  $ME(T) = E(T)$  for any tree  $T$  [11]. By using the quasi-order relation, it has also been obtained some results on extremal graphs with respect to matching energy among some classes of connected graphs with  $n$  vertices. For example, the extremal graphs in connected unicyclic, bicyclic graphs were determined by [11] and [13], respectively; the minimal graphs among connected  $k$ -cyclic ( $k \leq n - 4$ ) graphs and bipartite graphs were characterized by [14]; the maximal connected graph with given connectivity (resp. chromatic number) was determined by [15].

Let  $\mathcal{G}_{n,k}$  be the set of connected graphs of order  $n$  ( $\geq 2$ ) with edge connectivity  $k$  ( $1 \leq k \leq n - 1$ ). Let  $K_{n-1,1}^k$  be the graph, as shown in Fig. 1, obtained from  $K_1 \cup K_{n-1}$  by adding  $k$  edges between  $V(K_1)$  and  $V(K_{n-1})$ . In this paper, we show that  $K_{n-1,1}^k$  is the unique graph with maximum matching energy (resp. Hosoya index) in  $\mathcal{G}_{n,k}$ .

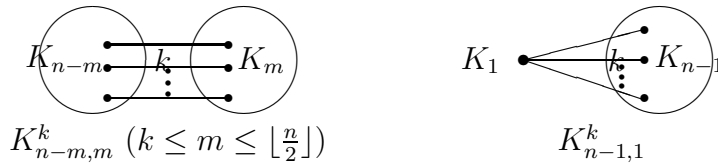


Fig. 1 Graphs  $K_{n-m,m}^k$  and  $K_{n-1,1}^k$ .

## 2 Main results

First we recall some notations. By  $\kappa'(G)$  and  $\delta(G)$ , we denote the edge connectivity and the minimum degree of a graph  $G$ , respectively. Let  $S$  be a nonempty proper subset of  $V$ . We use  $G[S]$  to denote the subgraph of  $G$  induced by  $S$ . An *edge cut* of  $G$ , denoted by  $\partial(S)$ , is a subset of  $E(G)$  of the form  $[S, \bar{S}]$ , where  $\bar{S} = V \setminus S$ . An edge

cut  $\partial(v)$  ( $v \in V$ ) is called a *trivial edge cut*. A *k-edge cut* is an edge cut of  $k$  elements. Let  $G \in \mathcal{G}_{n,k}$ . Then  $G$  must have a  $k$ -edge cut  $\partial(S)$  with  $1 \leq |S| \leq \lfloor \frac{n}{2} \rfloor$ .

**Lemma 2.1** *Let  $G \not\cong K_{n-1,1}^k$  be a graph in  $\mathcal{G}_{n,k}$  with a trivial  $k$ -edge cut. Then  $G \prec K_{n-1,1}^k$ .*

*Proof.* Let  $\partial(S)$  be a trivial  $k$ -edge cut of  $G$  with  $|S| = 1$ . Since  $G \not\cong K_{n-1,1}^k$ ,  $G[\bar{S}]$  is a proper subgraph of  $K_{n-1}$ . Hence  $G$  is a proper subgraph of  $K_{n-1,1}^k$ , and so the result follows from Lemma 1.3.  $\blacksquare$

**Lemma 2.2** *Let  $G \in \mathcal{G}_{n,k}$  be a graph without trivial  $k$ -edge cuts. Then for any  $k$ -edge cut  $\partial(S)$  of  $G$  with  $2 \leq |S| \leq \lfloor \frac{n}{2} \rfloor$ , we have  $|S| \geq k$ .*

*Proof.* For  $k \leq 2$ , the assertion is trivial, so suppose  $k \geq 3$ . Assume, to the contrary, that  $G$  has a  $k$ -edge cut  $\partial(S)$  with  $2 \leq |S| \leq k-1$ . By the facts that  $\delta(G) \geq \kappa'(G) = k$  and  $G$  has no trivial  $k$ -edge cuts, we have  $\delta(G) \geq k+1$ , and thus  $\sum_{v \in S} d_G(v) \geq |S|(k+1)$ . On the other hand,  $\sum_{v \in S} d_G(v) = 2e(G[S]) + k \leq |S|(|S|-1) + k$ . Therefore, we have  $|S|(k+1) \leq |S|(|S|-1) + k$ , that is,  $(|S|-1)(k-|S|) + |S| \leq 0$ , which is a contradiction. Therefore the result holds.  $\blacksquare$

For  $k \leq m \leq \lfloor \frac{n}{2} \rfloor$ , let  $K_{n-m,m}^k$  be the graph, as shown in Fig. 1, obtained from  $K_{n-m} \cup K_m$  by adding  $k$  independent edges between  $V(K_{n-m})$  and  $V(K_m)$ . It is easy to see that  $\kappa'(K_{n-m,m}^k) = k$  and  $\kappa'(K_{n-1,1}^k) = k$ .

We next show that for a graph  $G \in \mathcal{G}_{n,k}$  without trivial  $k$ -edge cuts,  $G \preceq K_{n-m,m}^k$  for some  $m$ . Before this, we introduce a new graph operation as follows.

let  $G_1$  be a graph in  $\mathcal{G}_{n,k}$  such that  $G_1$  has a  $k$ -edge cut  $\partial(S)$  with  $G_1[S] = K_m$ ,  $G_1[\bar{S}] = K_{n-m}$ , and  $k \leq m \leq \lfloor \frac{n}{2} \rfloor$ . Suppose that  $u_1, u_2 \in \bar{S}$ ,  $v_1, v_2 \in S$ ,  $e_1 = u_1v_1$ ,  $e_2 = u_1v_2$  are two edges of  $\partial(S)$ , and  $u_2$  is not incident with any edge in  $\partial(S)$ . If  $G_2$  is obtained from  $G_1$  by deleting the edge  $e_2$  and adding a new edge  $e'_2 = u_2v_2$ , we say that  $G_2$  is obtained from  $G_1$  by *Operation I*, as shown in Fig. 2. Clearly,  $G_2 \in \mathcal{G}_{n,k}$ .

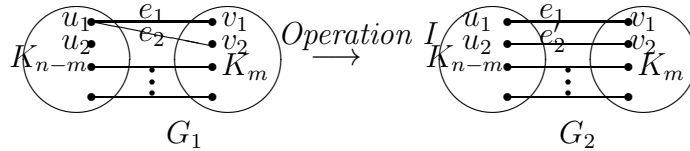


Fig. 2 The graphs  $G_1$  and  $G_2$  of  $\mathcal{G}_{n,k}$  in Operation I.

**Lemma 2.3** *If  $G_2$  is obtained from  $G_1$  by Operation I, then  $G_1 \prec G_2$ .*

*Proof.* By formula (1.6), we have

$$m(G_1, t) = m(G_1 - e_2, t) + m(G_1 - u_1 - v_2, t - 1),$$

and

$$m(G_2, t) = m(G_2 - e'_2, t) + m(G_2 - u_2 - v_2, t - 1).$$

Note that  $G_1 - e_2 \cong G_2 - e'_2$ , and  $G_1 - u_1 - v_2$  is isomorphic to a proper subgraph of  $G_2 - u_2 - v_2$ . So,  $m(G_1 - u_1 - v_2, t - 1) \leq m(G_2 - u_2 - v_2, t - 1)$  for all  $t$  and  $m(G_1 - u_1 - v_2, 1) < m(G_2 - u_2 - v_2, 1)$ . The result thus follows. ■

**Lemma 2.4** *Let  $G \in \mathcal{G}_{n,k}$  be a graph without trivial  $k$ -edge cuts. Then  $G \preceq K_{n-m,m}^k$  for some  $m$  with  $\max\{k, 2\} \leq m \leq \lfloor \frac{n}{2} \rfloor$ .*

*Proof.* Let  $\partial(S)$  be a  $k$ -edge cut of  $G$  with  $2 \leq |S| \leq \lfloor \frac{n}{2} \rfloor$ . Let  $|S| = m$ . Then  $m \geq k$  by Lemma 2.2. Let  $G_1$  be the graph obtained from  $G$ , by adding edges if necessary, such that  $G[S]$  and  $G[\bar{S}]$  are complete graphs. Therefore  $G \preceq G_1$  by Lemma 1.3. If  $G_1 \not\cong K_{n-m,m}^k$ , then by using Operation I repeatedly, we can finally get  $K_{n-m,m}^k$  from  $G_1$ . Hence  $G_1 \preceq K_{n-m,m}^k$  by Lemma 2.3. The proof is thus complete. ■

In the following, we show that  $K_{n-m,m}^k \prec K_{n-1,1}^k$  for  $m \geq 2$ .

**Lemma 2.5** *Suppose  $\max\{k, 2\} \leq m \leq \lfloor \frac{n}{2} \rfloor$ . Then  $e(K_{n-m,m}^k) < e(K_{n-1,1}^k)$ .*

*Proof.* Note that

$$e(K_{n-m,m}^k) = \frac{m(m-1)}{2} + \frac{(n-m)(n-m-1)}{2} + k,$$

and

$$e(K_{n-1,1}^k) = \frac{(n-1)(n-2)}{2} + k.$$

Hence we have

$$\begin{aligned} e(K_{n-1,1}^k) - e(K_{n-m,m}^k) &= \frac{n^2 - 3n + 2}{2} - \frac{n^2 + 2m^2 - 2mn - n}{2} \\ &= (m-1)(n-m-1) > 0. \end{aligned}$$

The proof is thus complete. ■

**Lemma 2.6** *Let  $m \geq 1$  be a positive integer. Then we have*

$$m(K_{m,m}^1, t) \leq m(K_{2m-1,1}^1, t) \text{ for all } t = 0, 1, \dots, m, \quad (2.1)$$

and

$$m(K_{m+1,m}^1, t) \leq m(K_{2m,1}^1, t) \text{ for all } t = 0, 1, \dots, m. \quad (2.2)$$

*Proof.* We apply induction on  $m$ . For  $m = 1$  and  $m = 2$ , the assertions are trivial since  $K_{2,2}^1$  and  $K_{3,2}^1$  are proper subgraphs of  $K_{3,1}^1$  and  $K_{4,1}^1$ , respectively. So suppose that  $m \geq 3$  and Ineqs. (2.1) and (2.2) hold for smaller values of  $m$ . By Lemma 1.2, we obtain that

$$\begin{aligned}
m(K_{m,m}^1, t) &= m(K_{m,m-1}^1, t) + (m-2)m(K_{m,m-2}^1, t-1) + m(K_m \cup K_{m-2}, t-1) \\
&= m(K_{m,m-1}^1, t) + (m-1)m(K_{m,m-2}^1, t-1) - m(K_{m-1} \cup K_{m-3}, t-2) \\
&= m(K_{m,m-1}^1, t) - m(K_{m-1} \cup K_{m-3}, t-2) + (m-1)[m(K_{m-1,m-2}^1, t-1) \\
&\quad + (m-1)m(K_{m-2,m-2}^1, t-2) - m(K_{m-3} \cup K_{m-3}, t-3)] \\
&\leq m(K_{m,m-1}^1, t) + (m-1)m(K_{m-1,m-2}^1, t-1) + (m-1)^2 m(K_{m-2,m-2}^1, t-2) \\
&\quad - m(K_{m-1} \cup K_{m-3}, t-2),
\end{aligned}$$

and

$$\begin{aligned}
m(K_{2m-1,1}^1, t) &= m(K_{2m-2,1}^1, t) + (2m-3)m(K_{2m-3,1}^1, t-1) + m(K_{2m-3}, t-1) \\
&= m(K_{2m-2,1}^1, t) + m(K_{2m-3}, t-1) + (2m-3)[m(K_{2m-4,1}^1, t-1) \\
&\quad + (2m-5)m(K_{2m-5,1}^1, t-2) + m(K_{2m-5}, t-2)] \\
&\geq m(K_{2m-2,1}^1, t) + (2m-3)m(K_{2m-4,1}^1, t-1) \\
&\quad + (2m-3)(2m-5)m(K_{2m-5,1}^1, t-2).
\end{aligned}$$

By the induction hypothesis, we obtain that

$$\begin{aligned}
m(K_{m,m-1}^1, t) &\leq m(K_{2m-2,1}^1, t), \\
m(K_{m-1,m-2}^1, t-1) &\leq m(K_{2m-4,1}^1, t-1), \\
m(K_{m-2,m-2}^1, t-2) &\leq m(K_{2m-5,1}^1, t-2).
\end{aligned}$$

Since  $m \geq 3$ , we have that  $m-1 \leq 2m-3$  and  $(m-1)^2 \leq (2m-3)(2m-5)$  when  $n \geq 4$ . Notice that for  $m = 3$ ,  $K_{m-2,m-2}^1 = K_{m-1} \cup K_{m-3}$ , and  $(m-1)^2 - 1 = (2m-3)(2m-5)$ . Hence Ineq. (2.1) holds.

By Lemma 1.2, we get that

$$\begin{aligned}
m(K_{m+1,m}^1, t) &= m(K_{m,m}^1, t) + (m-1)m(K_{m-1,m}^1, t-1) + m(K_{m-1} \cup K_m, t-1) \\
&\leq m(K_{m,m}^1, t) + m \cdot m(K_{m-1,m}^1, t-1) \\
&= m(K_{m,m}^1, t) + m \cdot [m(K_{m-1,m-1}^1, t-1) \\
&\quad + (m-2)m(K_{m-1,m-2}^1, t-2) + m(K_{m-1} \cup K_{m-2}, t-2)] \\
&\leq m(K_{m,m}^1, t) + m \cdot [m(K_{m-1,m-1}^1, t-1) \\
&\quad + (m-1)m(K_{m-1,m-2}^1, t-2)] \\
&= m(K_{m,m}^1, t) + m \cdot m(K_{m-1,m-1}^1, t-1) + m(m-1)m(K_{m-1,m-2}^1, t-2),
\end{aligned}$$

and

$$\begin{aligned}
m(K_{2m,1}^1, t) &= m(K_{2m-1,1}^1, t) + (2m-2)m(K_{2m-2,1}^1, t-1) + m(K_{2m-2}, t-1) \\
&= m(K_{2m-1,1}^1, t) + m(K_{2m-2}, t-1) + (2m-2)[m(K_{2m-3,1}^1, t-1) \\
&\quad + (2m-4)m(K_{2m-4,1}^1, t-2) + m(K_{2m-4}, t-2)] \\
&\geq m(K_{2m-1,1}^1, t) + (2m-2)m(K_{2m-3,1}^1, t-1) \\
&\quad + (2m-2)(2m-4)m(K_{2m-4,1}^1, t-2).
\end{aligned}$$

By the induction hypothesis and Ineq. (2.1), we have that

$$\begin{aligned}
m(K_{m,m}^1, t) &\leq m(K_{2m-1,1}^1, t) \\
m(K_{m-1,m-1}^1, t-1) &\leq m(K_{2m-3,1}^1, t-1) \\
m(K_{m-1,m-2}^1, t-2) &\leq m(K_{2m-4,1}^1, t-2).
\end{aligned}$$

Notice that  $m \leq 2m-2$  and  $m(m-1) \leq (2m-2)(2m-4)$ . Therefore Ineq.(2.2) holds.

The proof is thus complete. ■

**Lemma 2.7** Suppose  $2 \leq m \leq \lfloor \frac{n}{2} \rfloor$ . Then

$$m(K_{n-m,m}^1, t) \leq m(K_{n-1,1}^1, t) \text{ for all } t = 0, 1, \dots, \lfloor \frac{n}{2} \rfloor.$$

*Proof.* We apply induction on  $n$ . As the two cases  $n = 2m$  and  $n = 2m+1$  were proved by Lemma 2.6, we proceed to the induction step. By Lemma 1.2 and the induction hypothesis, we have that

$$\begin{aligned}
m(K_{n-m,m}^1, t) &= m(K_{n-m,m-1}^1, t) + (m-2)m(K_{n-m,m-2}^1, t-1) + m(K_{n-m} \cup K_{m-2}, t-1) \\
&\leq m(K_{n-m,m-1}^1, t) + (m-1)m(K_{n-m,m-2}^1, t-1) \\
&\leq m(K_{n-2,1}^1, t) + (m-1)m(K_{n-3,1}^1, t-1) \\
&= m(K_{n-2}, t) + m(K_{n-3}, t-1) \\
&\quad + (m-1)(m(K_{n-3}, t-1) + m(K_{n-4}, t-2)) \\
&= m(K_{n-2}, t) + m \cdot m(K_{n-3}, t-1) + (m-1)m(K_{n-4}, t-2),
\end{aligned}$$

and

$$\begin{aligned}
m(K_{n-1,1}^1, t) &= m(K_{n-1}, t) + m(K_{n-2}, t-1) \\
&= m(K_{n-2}, t) + (n-2)m(K_{n-3}, t-1) \\
&\quad + m(K_{n-3}, t-1) + (n-3)m(K_{n-4}, t-2) \\
&= m(K_{n-2}, t) + (n-1)m(K_{n-3}, t-1) + (n-3)m(K_{n-4}, t-2).
\end{aligned}$$

Thus the result follows by the fact  $m \leq n-2$ . ■

**Lemma 2.8** Suppose  $k \leq m \leq \lfloor \frac{n}{2} \rfloor$ . Then

$$m(K_{n-m,m}^k, t) \leq m(K_{n-1,1}^k, t) \text{ for all } t = 0, 1, \dots, \lfloor \frac{n}{2} \rfloor.$$

*Proof.* We apply induction on  $k$ . As the case  $k = 1$  was proved by Lemma 2.7, we suppose that  $k \geq 2$  and the assertion holds for smaller values of  $k$ . By formula (1.6), we have that

$$m(K_{n-m,m}^k, t) = m(K_{n-m,m}^{k-1}, t) + m(K_{n-m-1,m-1}^{k-1}, t-1),$$

and

$$m(K_{n-1,1}^k, t) = m(K_{n-1,1}^{k-1}, t) + m(K_{n-2}, t-1).$$

By the induction hypothesis and Lemma 1.3, we obtain that  $m(K_{n-m,m}^{k-1}, t) \leq m(K_{n-1,1}^k, t)$  and  $m(K_{n-m-1,m-1}^{k-1}, t-1) \leq m(K_{n-2}, t-1)$ . Thus the result follows. ■

Together with Lemmas 2.5 and 2.8, we directly obtain the following result.

**Corollary 2.9** *Suppose  $\max\{k, 2\} \leq m \leq \lfloor \frac{n}{2} \rfloor$ . Then  $K_{n-m,m}^k \prec K_{n-1,1}^k$ .*

**Theorem 2.10** *Let  $G$  be a graph in  $\mathcal{G}_{n,k}$ . Then  $ME(G) \leq ME(K_{n-1,1}^k)$ . The equality holds if and only if  $G \cong K_{n-1,1}^k$ .*

*Proof.* Notice that  $K_{n-1,1}^k \in \mathcal{G}_{n,k}$ . Let  $G \not\cong K_{n-1,1}^k$  be a graph in  $\mathcal{G}_{n,k}$ . It suffices to show that  $G \prec K_{n-1,1}^k$ . If  $G$  has a trivial  $k$ -edge cut, then we have  $G \prec K_{n-1,1}^k$  by Lemma 2.1. Otherwise, by Lemma 2.4 and Corollary 2.9, we obtain that  $G \prec K_{n-1,1}^k$  again. The proof is thus complete. ■

By the proof of Theorem 2.10 and the definition of Hosoya index, we can get the following result on Hosoya index.

**Theorem 2.11** *Let  $G$  be a graph in  $\mathcal{G}_{n,k}$ . Then  $Z(G) \leq Z(K_{n-1,1}^k)$ . The equality holds if and only if  $G \cong K_{n-1,1}^k$ .*

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